MATH10222, Calculus & Applications : Examples 1

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Questions marked with an asterisk are to be handed in to your supervisor.

Q1. **[Chapter 1, section 1]**[1] In a Cartesian coordinate system with basis vectors $\{\underline{i}, \underline{j}, \underline{k}\}$, the position vector of a particle P at a time t is given by

$$\underline{r} = (2t^2 - 5t)\underline{i} + (4t + 2)\underline{j} + t^3\underline{k}\,.$$

Find the corresponding velocity and acceleration of P at time t. Deduce the speed of P and a unit vector tangent to the path of P at the point where $t = 2$.

If P has a mass m, give the linear momentum $\underline{p}$.

Q2*. **[Chapter 1, example 1.1]** In a Cartesian coordinate system with basis vectors $\{\underline{i}, \underline{j}, \underline{k}\}$, the acceleration vector of a particle P at a time t is given by

$$\underline{a} = 6t\underline{i} + 10\underline{k}\,.$$

Find the velocity and position vectors of P at time t, given that, initially, P is at the origin, and moving with a velocity $-2\underline{j} + \underline{k}$.

Q3*. **[Chapter 1, section 1]** A particle P moves with constant speed u in the anti-clockwise direction around a circle of radius b and centre O in the (x, y) plane. At time $t = 0$, P is at the point $(b, 0)$. Show that the position vector of P at time t is given by

$$\underline{r} = b\cos(ut/b)\underline{i} + b\sin(ut/b)\underline{j},$$

where $\underline{i}$ and $\underline{j}$ are the Cartesian basis vectors in the directions of increasing x and y respectively.

Given the above position vector, determine the velocity and acceleration vectors for P at time t. Verify that $\ddot{\underline{r}} = -\omega^2\underline{r}$, where $\omega = u/b$.

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[1] These are references to parts of the lecture notes or other examples sheet questions that you may find useful when constructing your answer.

Q4. [Chapter 1, section 5] Starting from the velocity formula in plane-polar coordinates (in the usual notation):

$$\underline{v} = \dot{r}\hat{\underline{r}} + r\dot{\theta}\hat{\underline{\theta}}\,,$$

differentiate with respect to time to derive the corresponding acceleration formula.

Show that this acceleration formula reduces to that found above in question 3 when the motion is around a circle of radius b at a constant speed u.

Hint:

You should remember that the unit vectors $\hat{\underline{r}}$ and $\hat{\underline{\theta}}$ are NOT constants and must be differentiated too! If you can't remember how/why – see the section referenced in the lecture notes.

Q5*. [Chapter 1, example 7.3] A uniform (thin) rod of total mass M and length $2a$ lies along the interval $[-a, a]$ of the x-axis. A particle P of mass m is situated at the point $x = b > a > 0$. Find the gravitational force exerted on the particle P by the rod.

Hint:

Since everything is along a line, this problem is rather simpler than the example we did in the lecture notes.

You should try the questions without looking here first ... but some 'answers' are as follows :

Q1: $\underline{a} = 4\,\underline{i} + 6t\,\underline{k}$.
Q2: $\underline{r} = t^3\,\underline{i} - 2t\,\underline{j} + (5t^2 + t)\,\underline{k}$.
Q5:

$$\underline{F} = -\frac{mMG}{b^2 - a^2}\,\underline{i}\,.$$

Full solutions will be provided after the problems have been discussed in the supervision classes.
