MATH10222, REVISION!

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This course relies upon you knowing the material that was covered in MATH10121 'Calculus and vectors' (semester 1) and in the first half of this course (on the solution of ODEs).

In particular, some **very basic** facts that you need to know are:

- For vectors $\underline{a}$ and $\underline{b}$,
  - The 'dot product' operator is defined to be

    $$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta \,.$$

  - The 'cross product' operator is defined to be

    $$\underline{a} \wedge \underline{b} \equiv \underline{a} \times \underline{b} = |\underline{a}||\underline{b}| \sin\theta \, \hat{\underline{n}} \,.$$

  - Here $\hat{\underline{n}}$ is a unit vector (ie., $|\hat{\underline{n}}| = 1$) perpendicular to both $\underline{a}$ and $\underline{b}$ such that $(\underline{a}, \underline{b}, \hat{\underline{n}})$ forms a right-handed set, and θ is the angle between $\underline{a}$ and $\underline{b}$.
  - Note that I will use the notation $\wedge$ to indicate a cross product, but some people use $\times$. They are the same in this context.
  - If $\underline{i}, \underline{j}, \underline{k}$ form a right-handed set of Cartesian basis vectors with

    $$\underline{a} = \alpha \underline{i} + \beta \underline{j} + \gamma \underline{k} \,,$$

    and

    $$\underline{b} = x \underline{i} + y \underline{j} + z \underline{k} \,,$$

    then

    $$\underline{a} \wedge \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha & \beta & \gamma \\ x & y & z \end{vmatrix} = \begin{pmatrix} \beta z - \gamma y \\ \gamma x - \alpha z \\ \alpha y - \beta x \end{pmatrix} \,.$$

    We could have obtained this same result by expanding using:

    $$\underline{i} \wedge \underline{j} = \underline{k} \,, \quad \underline{i} \wedge \underline{k} = -\underline{j} \,, \quad \underline{j} \wedge \underline{k} = \underline{i} \,,$$

    with other combinations following from the **general** results

    $$\underline{c} \wedge \underline{c} = \underline{0} \,, \quad \underline{c} \wedge \underline{d} = -\underline{d} \wedge \underline{c} \,,$$

    for any two vectors $\underline{c}$ and $\underline{d}$.

- If we have three vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$, then

$$\underline{a} \wedge (\underline{b} + \underline{c}) = \underline{a} \wedge \underline{b} + \underline{a} \wedge \underline{c} \,.$$

  There is also the 'scalar triple product':

$$(\underline{a} \wedge \underline{b}) \cdot \underline{c} = (\underline{b} \wedge \underline{c}) \cdot \underline{a} = (\underline{c} \wedge \underline{a}) \cdot \underline{b} \,.$$

  If $\underline{a}$, $\underline{b}$ and $\underline{c}$ are co-planar, then the scalar triple product is zero. We also note that

$$(\underline{a} \wedge \underline{b}) \cdot \underline{b} = 0 \,, \quad \text{and} \quad (\underline{a} \wedge \underline{b}) \cdot \underline{a} = 0 \,.$$

- Given a scalar $\mu(t)$ and vectors $\underline{b}(t)$, $\underline{c}(t)$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mu \underline{b}) = \dot{\mu} \underline{b} + \mu \dot{\underline{b}} \,,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{b} \cdot \underline{c}) = \dot{\underline{b}} \cdot \underline{c} + \underline{b} \cdot \dot{\underline{c}} \,,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{b} \wedge \underline{c}) = \dot{\underline{b}} \wedge \underline{c} + \underline{b} \wedge \dot{\underline{c}} \,,$$

  all follow from the product rule.

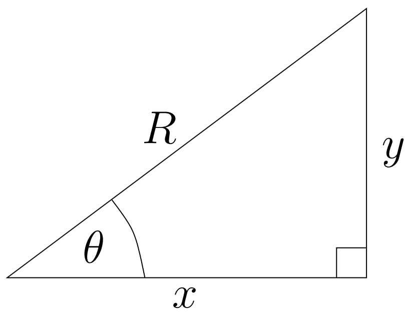

Figure 1: Right-angled triangles.

- For the triangle shown, we have that

$$x = R \cos \theta \,,$$

  and

$$y = R \sin \theta \,.$$

Greek alphabet

We will make use of some of the following symbols:

α	A	Alpha
β	B	Beta
γ	Γ	Gamma
δ	Δ	Delta
ϵ	E	Epsilon
ζ	Z	Zeta
η	H	Eta
θ	Θ	Theta
ι	I	Iota
κ	K	Kappa
λ	Λ	Lambda
μ	M	Mu
ν	N	Nu
ξ	Ξ	Xi
o	O	Omicron
π	Π	Pi
ρ	P	Rho
σ	Σ	Sigma
τ	T	Tau
υ	Υ	Upsilon
ϕ	Φ	Phi
χ	X	Chi
ψ	Ψ	Psi
ω	Ω	Omega